Performance measures Adjusted for the Risk Situation (PARS) – replacing the PV?

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1 Why PARS?

2 Performance measures Adjusted for the Risk Situation (PARS)

3 Examples
   - Performance benchmarking
   - An individual saving for retirement

4 Dynamic control for a sovereign debt manager

5 Conclusion
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PARS in one sentence

PARS shall measure the financial performance in the actual quantity of interest, thereby taking into account the individual (risk) situation.
Present value (PV) and risk situation

Figure: Price time series of bonds with a time to maturity of 30 years (orange) and 2 years (blue dashed) in 2016, rebased to 100 on Jan-01.

Questions
- Which portfolio is more risky?
- Why?
- And if you are a debt manager interested in projectable payments?
- Why?
- And if you want to build a new home this year?
- Why?
Present value (PV) and risk situation

Figure: Price time series of bonds with a time to maturity of 30 years (orange) and 2 years (blue dashed) in 2016, rebased to 100 on Jan-01.

Questions
- Which portfolio is more risky? Why?
**Why PARS?**

**Present value (PV) and risk situation**

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- And if you are a debt manager interested in projectable payments?

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(6)
Present value (PV) and risk situation

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- And if you want to build a new home this year? Why?

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Why PARS?

Risk situation

- In the situation of a typical debt manager, issuing long-term bonds is considered low risk.
- In the situation of a typical home buyer, investing in long-term bonds (or stocks) is risky.

Risk depends on the situation!

Your risk perception depends on your situation, put in financial terms: on your future (external) cash flows.

Judging risk based on the present value can be misleading!

Objective

Find new key figure reflecting your risk situation, in order to replace PV!
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A first PARS: Asset manager with limited-time mandate

Consider an asset manager at time $t$ with a limited-time mandate, whose assets will be withdrawn at time $T > t$.

Key indicator of interest: the fund’s PV forward at $T$

For the portfolio PV $V_t(P^t)$ and the price $P_{t,T}$ of a (default-risk-free) zero-coupon bond maturing at $T$, we define

$$\Pi_t = \frac{V_t(P^t)}{P_{t,T}}.$$  \hfill (1)

Risk-free benchmark strategy

Given the situation of the asset manager, the risk-free benchmark strategy is to put all the assets into the zero bond with value $P_{t,T}$: Indeed, $\Pi$ is constant under this strategy.
Performance measures Adjusted for the Risk Situation (PARS)

PARS Π: The generic solution

Definition (PARS = quantity of interest = level of consumption)

\[ \Pi_t := \frac{V_t(P^t) - V_t(C)}{V_t(E)} \]  

- \( V_t(P^t) \) present value (PV) of portfolio at time \( t \)
- \( V_t(C) \) PV of inevitable cost cash flows (e.g., transaction-independent fees)
- \( V_t(E) \) > 0 cost PV of one unit of preferred consumption / payout (e.g., terminal payout \( E := 1_T \))
- \( P^t, C, E \) adapted stochastic processes of cash flows in \( \mathbb{R} \)

Summary

PARS shall cast the portfolio PV into the quantity of interest to the owner.
Performance measures Adjusted for the Risk Situation (PARS)

**PARS: Define what is risk-free**

**Interpretation as (hypothetical) trade**

Sell the portfolio, replacing it by a combination of $C$ and $E$ with the same present value:

$$V_t(P^t) = V_t(C) + xV_t(E) = V_t(C + xE) \quad (3)$$

The solution $x = x(t) \in \mathbb{R}$ of (3) is equal to $\Pi_t$.

We call the portfolio $C + \Pi_tE$ in (3) also the *benchmark strategy*.

**Proposition (Benchmark strategy is risk-free in $\Pi$)**

$\Pi_{t+\Delta} = \Pi_t$ for all $\Delta > 0$ if the benchmark strategy is implemented in $t$.

**Proof:** Insert $V_{t+\Delta}(P^{t+\Delta}) = V_{t+\Delta}(C + \Pi_tE) = V_{t+\Delta}(C) + \Pi_tV_{t+\Delta}(E)$ into (2) for $t + \Delta$. \qed
In the example of the asset manager, the risk horizon is $T > t$. How to get an idea about relevant key properties of the portfolio at $t$?

- Without PARS: Simulate market and portfolio evolution until $T$
- PARS are already forward at $T$ – no simulation needed, only market-standard valuation rules!

**Instantaneous risk evaluation**
Using PARS, the relevant risk can be computed in an instantaneous evaluation (e.g., using a sensitivity together with a covariance).
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Performance benchmark: A well-known example of PARS

Take a (self-financing) index $I$, set $C := 0$ and $E := 1_T I_T$ in

$$\Pi_t = \frac{V_t(P^t) - V_t(C)}{V_t(E)} = \frac{V_t(P^t)}{I_t}$$

(Non-annualized) return of the PARS (4) from $t = 0$ to $t = T$ is equal to

$$\log \left( \frac{\Pi_T}{\Pi_0} \right) = (\log V_T(P^T) - \log V_0(P^0)) - (\log I_T - \log I_0).$$

Conclusion

The return of the PARS (4) is the overperformance over the benchmark $I$.

Tracking error of index funds = standard deviation of increments of (4)

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Note that $I_t = V_t(1_T I_T)$ for $T > t$ assuming standard valuation.
An individual saving for retirement

- Main uncertainties: unemployment, death, and inflation
- Constant yearly salary $s$ until (stopping) time of retirement $\tau_R$ (or time of final unemployment)
- Basic needs: $c \cdot I_t$ per year until (stopping) time of death $\tau_D$ ($I_t$ inflation index)
- Excess money: spend on evenly distributed luxury consumption

**PARS components for the individual**

$$V_t(C) = \mathbb{E}^t \left[ \sum_{r:t<r\leq\tau_D} B(t)/B(r)cl_r \right] - s\mathbb{E}^t \left[ \sum_{r:t<r\leq\tau_R} B(t)/B(r) \right]$$

$$V_t(E) = \mathbb{E}^t \left[ \sum_{r:t<r\leq\tau_D} B(t)/B(r) \right]$$

for the discrete bank account $B(t)$. 
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Dynamic control for a sovereign debt manager

Risk situation of a sovereign debt manager

- Government will always be a net debtor
- Relatively stable tax earnings
- Fixing debt expenses for the whole future is considered risk-free

Assumptions:
- There is no government default (!)
- Long-term interest rate is positive (!)

PARS for our debt manager

Set $C :\equiv 0$ and $E :\equiv 1_{(0,\infty)}$ (a perpetuity, i.e., an instrument promising a continuous cash flow stream of 1 until $\infty$) in

$$
\Pi(t) = \frac{V_t(P^t) - V_t(C)}{V_t(E)} = \frac{V_t(P^t)}{p^\infty(t)}
$$

with the value of the perpetuity $V_t(E) = p^\infty(t) :\equiv \int_0^\infty P(t, t + s)ds$. 
Debt manager: Self-financing dynamics

Changing to simpler continuous-time notation!

- Use a simple arbitrage-free Gaussian dynamic term-structure model (see [HPS13]) with an n-dimensional Wiener process $W$ subject to a volatility matrix $\sigma \in \mathbb{R}^{n \times n}$
- Under such an arbitrage-free DTSM, any tradable instrument with non-zero present value $Q$ has the dynamics

$$dQ(t) = Q(t) \left( r(t) dt + \psi_Q^T(t) \sigma b \ dt + \psi_Q^T(t) \sigma \ dW(t) \right)$$

for the short rate $r$, the market price of risk $b$ and the (relative) factor sensitivity $\psi_Q$

- Generalize this to the present value $V$ of a self-financing portfolio:

$$dV(t) = V(t) \left( r(t) dt + \psi^T(t) \sigma b \ dt + \psi^T(t) \sigma \ dW(t) \right)$$

($V < 0$ for a debt portfolio)
Dynamics with external cash flows

- Add canonical (PARS-implied) external cash flow stream \( \kappa(t) = -V(t)/p^\infty(t) \):

\[
dV(t) = V(t) \left( r(t) dt + \psi^\top(t) \sigma b \, dt + \psi^\top(t) \sigma \, dW(t) \right) + \kappa(t) \, dt
\]

- Apply Itô’s formula to obtain the PARS dynamics

\[
d\Pi(t) = \Pi(t) \left( (\psi(t) - \psi^\infty(t))^\top \sigma \xi(t) \, dt + (\psi(t) - \psi^\infty(t))^\top \sigma \, dW(t) \right)
= \Pi(t) \left( \Delta^\top(t) \sigma \xi(t) \, dt + \Delta^\top(t) \sigma \, dW(t) \right)
\]

where \( \xi(t) = b - \sigma^\top \psi^\infty(t) \).

Perpetuity risk-free

Choosing \( \Delta \equiv 0 \) (sensitivity of the perpetuity) is the risk-free strategy.
Stochastic control problem

$$\Delta = \text{Difference of relative sensitivity to (relative) sensitivity of perpetuity}$$

Stochastic control problem

Find optimal admissible control process $\Delta$ maximizing

$$\mathbb{E}^{0,\pi,u}[U(\Pi^\Delta(T))]$$

for some concave increasing utility function $U(\pi) = -(-\pi)^\gamma$ for $\gamma > 1$ and $\pi < 0$.

Optimal control (through solution of HJB equation)

$$\Delta^*(t) \equiv \frac{1}{1-\gamma} \sigma^{-\top} \xi,$$

assuming constant adjusted market price of risk $\xi$. 
Optimal solution in risk-return graph

Figure: Risk-return graphs of selected strategies in present value $V$ (left) and in PARS $\Pi$ (right), incl. (near-)optimal HJB solution. In green the portfolios with optimal sensitivity under $\Pi$, in blue those with optimal sensitivity under $V$, in red the stable portfolios with different initial maturities. Note that $V < 0$. Term-structure model and data from [HPS13].
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PARS: a new risk paradigm

- Risk perception depends on situation, i.e., future cash flows incl. (financial) consumption preferences
- Risk transformation: As PARS shall measure the actual quantity of interest, their variability exhibits the relevant risk.
- Using a standard utility function on PARS allows to capture aversion to the relevant risk.
- Instantaneous risk evaluation:
  - Using PARS, the relevant risk can be computed in an instantaneous evaluation (e.g., using a sensitivity together with a covariance).
  - No simulation of present values necessary! Market evolution models are replaced by market-standard valuation models.

Thank you for your attention!

