Theorem (by M. Sperl from the existence and uniqueness of the woman of one's dreams)

For every man there exists a unique woman of his dreams.

Proof: (by A. Niedermeier)

i) Let $\mathbb{W}$ be the set of all women and $m \in \mathbb{M}$ be a man. Further let $\varphi_m : \mathbb{W} \to \mathbb{R}$ be a function, the specific function of m. Because of $|\mathbb{W}| < \infty$ we can consider $\mathbb{W}$ without loss of generality under the discreet topology $\tau = \mathcal{P}(\mathbb{W})$. Thereby φ_m is continuous. On the other hand $\mathbb{W}$ as a finite set is compact (according to τ). So $\varphi_m(\mathbb{W})$ is compact in $\mathbb{R}$ as continuous image of a compact set, and as such limited and φ_m has a maximum on $\mathbb{W}$.

ii) Let $w_1, w_2 \in \mathbb{W}, \varphi_m(w_1) = \varphi_m(w_2) = \max\limits_{w \in \mathbb{W}} \varphi_m(w), w_1 \neq w_2$. The application leads us to consider the following two cases:

1st case: m decides for a single $w_i, i \in \{1,2\}$, without loss of generality for w_1. Then it is tivial to see $\varphi_m(w_1) \gneqq \varphi_m(w_2)$ #

2nd case: m can't decide for either. Then normally $\max\big(\varphi_m(w_1), \varphi_m(f_2)\big) \ll \max\limits_{w \in \mathbb{W}} \varphi_m(w)$ $\big($by the influence of w_i on $\varphi_m(w_i)\big)$ # $\square$

Remark 1: In praxis we have to notice, that $|\mathbb{W}|$ is very big. Furthermore $\varphi_m(w)$ is mostly difficult to determine even for a single w.

Proposition & definition: The function given by $\Phi : \mathbb{M} \to \mathbb{W}, m \mapsto \arg\max\limits_{w \in \mathbb{W}} \varphi_m(w)$ is welldefined. It is called obligatory function on $\mathbb{M}$.

Remark 2: By $|\mathbb{W}| \neq |\mathbb{M}|$ we can see, that no bijection between $\mathbb{F}$ and $\mathbb{M}$ can exist. Generaly Φ is neither injective nor surjective.

Remark 3: Analogously the dual statement for $\mathbb{M}, w \in \mathbb{W}$ and ψ_w can be shown. This leads with the dual function Ψ and remark 2 to the problem, that $\Phi \circ \Psi \neq id_{\mathbb{W}}, \Psi \circ \Phi \neq id_{\mathbb{M}}$. For a solution (on maximum subsets of $\mathbb{W}$ and $\mathbb{M}$) we have to generalize the functions φ_m and ψ_w, what means lower values on $\mathbb{R}$ for the maximizing on generally nonempty subsets of $\mathbb{W}$ and $\mathbb{M}$.

Remark 4: For a generalized model one can consider $\mathbb{M}$ and $\mathbb{W}$ depending on time t. In reality mostly (meaning for fixed $t_0 \in \mathbb{R}$) a maximum of φ_m (resp. ψ_w) on $\mathbb{W}(t_0) \cap \mathbf{P}_m(t_0)$ with $\mathbf{P}_m(t_0) \subset \mathbb{P}(t_0) := \mathbb{M}(t_0) \cup \mathbb{W}(t_0)$ $\big($resp. on $\mathbb{M}(t_0) \cap \mathbf{P}_w(t_0)\big)$ is searched once $\mathbb{W}(t_0) \cap \mathbf{P}_m(t_0) \neq \emptyset$ $\big($resp. $\mathbb{M}(t_0) \cap \mathbf{P}_w(t_0) \neq \emptyset\big)$. Thereby trivially $m \in \mathbf{P}_m(t_0)$ $\big($or $w \in \mathbf{P}_w(t_0)\big)$, what means $\mathbf{P}_m(t_0)$ is a neighbourhood of m according to $\tau' := \mathcal{P}(\mathbb{P}) \cong \tau \times \mathcal{P}(\mathbb{W})$ $\big($resp. $\mathbf{P}_w(t_0)$ of $w\big)$.

Remark 5: (by T. Voigtmann)

Numercally the decision for a $w_i \in \mathbb{F}$ can be done by a conjugate gradient method ("two sided attraction") minimizing $-\varphi_m$. In this case it is possible, that m is caught in a local minimum of $-\varphi_m$. Furthermore it has to be considered, that a extension of φ_m on the closure $\overline{\mathbb{W}}$ is needed for the gradient being welldefinded. Generally $\tilde{w} := \arg\max_{w \in \overline{\mathbb{W}}} \varphi_m(w) \notin \mathbb{W}$ holds.

Remark 6: (by T. Voigtmann)

To be more exact the follwoing differential equation holds for timedependend φ_m:

$$\dot{\varphi}_m(f_i) = \underbrace{P(f_i)}_{\text{starting-value}} + \underbrace{\partial_j \varphi_m(f_k)\varepsilon_{ijk}}_{\text{Influence of the other } \varphi_m(f_j) \text{ on } \varphi_m(f_i)} + \underbrace{D^{ij}\varphi_m(f_j)}_{\text{dilemmatensor}}$$

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